**Solutions to Problem Set 1**

1.22 c

$\overline{Y} = 93.7$

$S = 9.5537$

1.22 d

| st dev | $\pm 1$ | $\pm 2$ | $\pm 3$ |
|---|---|---|---|
| within | $84.1 \to 103.3$ | $74.6 \to 112.8$ | $65.03 \to 122.3$ |
| num within | 13 | 20 | 20 |
| num expected | $\sim 13$ | $\sim 19$ | $\sim 20$ |

2.9 a

$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1$

$.15 + .15 + .4 + 3P(E_5) = 1$

$P(E_5) = .1$

$P(E_4) = .2$

2.9 b

$.3 + .1 + 3P(E_3) = 1$

$3P(E_3) = .6$

$P(E_3) = P(E_4) = P(E_5) = .2$

2.10 a

$E_1 =$ Turn Left $E_2 =$ Turn Right $E_3 =$ Straight

2.10 b

$P(Turn) = P(E_1 \vee E_2) = P(E_1) + P(E_2) = 1/3 + 1/3 = 2/3$

2.12 a

$P(Needsglasses) = P(Needsglasses \wedge Reads) + P(Needsglasses \wedge doesnotread) = .44 + .14 = .58$

2.12 b

$P(\sim glasses \wedge reads) = .14$

2.12 c

$P(uses) = P(uses \wedge needs) + P(uses \wedge \sim needs) = .44 + .02 = .46$

2.13 a
$P(hit1st \wedge miss2nd) = P(hit1st) * P(miss2nd) = .1 * .9 = .09$
2.13 b
$1 - P(E_4) = .19$

2.19 a
$\{LL, LR, LS, SL, SR, SS, RL, RR, RS\}$ where $LR =$ first car left, second car right
$||samplespace|| = 9$
2.19 b
5/9
2.19 c
$1 - P(SS) = 5/9$

2.33 $9 * 10^6 = 9000000$

2.35 ${}_9C_3 *_6 C_5 * 1 = \frac{9*8*7}{3*2*1} * 6 * 1 = 3 * 4 * 7 * 6 = 504$

2.41 a
$4/50 * 3/49 * 2/48 = \frac{24}{117600}$
2.41 b
note: 3 different possible prize combos $\{(1st, 3rd), (1st, 2nd), (2nd, 3rd)\}$
$\frac{3*12*46}{50*49*48} = \frac{1656}{117600}$
2.41 c
$\frac{3*4*46*45}{50*49*48} = 24840/117600$
2.41 d
$\frac{46*45*44}{117600} = 91080/117600$

2.61
$P(A) = .4$
$P(B) = .37$
$P(AB) = .1$

$P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - /33 = .67$
$P(\overline{A}) = .6$
$P(\overline{A \cup B}) = .33$
$P(\overline{AB}) = .9$
$P(A|B) = \frac{P(AB)}{P(B)} = \frac{.1}{.37} = .2703$
$P(B|A) = \frac{P(AB)}{P(A)} = \frac{.1}{.4} = .25$

2.86

$P(notdefect) = P(ND|1) * P(1) + P(ND|2) * P(2) = .92 * .4 + .9 * .6 = .9080$

2.87

$P(buys) = P(buys|sees) * P(sees) + P(buys| \sim see) * P(\sim see) = 1/3 * (.02 + .2 - .01) + .1 * (1 - (.02 + .2 - .01)) = .149$

2.90 a
$P(bothlie) = .1 * .95 = .095$
2.90 b
$P(guiltpos, innocentneg) = .9 * .95 = .855$
2.90 c
$P(guiltyneg, innocentpos) = .05 * .1 = .005$
2.90 d
$P(eithororbothpos) = 1 - P(bothneg) = 1 - .9 * .05 = .955$

2.99
$P(sick|positive) = \frac{P(sick \wedge positive)}{P(positive)} = \frac{.9 * 01}{.99 * .1 + .01 * .9} = .0833$

2.103
$P(male|neg) = \frac{P(male \wedge neg)}{P(neg)} = P(neg|male) * P(male)/P(neg) = \frac{.6 * .25}{.6 * .25 + .3 * .75} = .4$

3.2
$P(W = -1) = .5$
$P(W = 1) = .25$
$P(W = 2) = .25$

3.10
$E(Y) = .4 * 1 + .3 * 2 + .2 * 3 + .1 * 4 = 2$
$E(1/Y) = .4 * 1 + .3 * \frac{1}{2} + .2 * \frac{1}{3} + .1 * \frac{1}{4} = .6417$
$E(Y^2 - 1) = .4 * 0 + .3 * 3 + .2 * 8 + .1 * 15 = 4$
$Var(Y) = .4 * 1 + .3 * 0 + .2 * 1 + .1 * 4 = 1$

3.13
want: $50 = E(Premium) - 15$
so: $E(Premium) = 65$
so: $.02(C - 1000) + .98 * C = 65 \rightarrow C = 85$

3.28 a
$.8^{14} * .2^6 *_{20} C_{14} = .1091$
3.28 b
$\sum_{i=10}^{20} .8^i * .2^{20-i} *_{20} C_i = .9994$
3.28 c
$\sum_{i=14}^{18} .8^i * .2^{20-i} *_{20} C_i = .8441$
3.28 d
$\sum_{i=0}^{16} .8^i * .2^{20-i} *_{20} C_i = .5886$

3.40
$E(N) = \sum_{N=0}^{10} N * \Pr(N) = \sum_{N=0}^{10} N * .1^N .9^{10-N} {}_{10}C_N = 1$
$Var(N) = \sum_{N=0}^{10} (N-1)^2 * \Pr(N) = .9$

3.41
$E(TC) = 20000 + \sum_{N=0}^{10} ((N * 30000) + (10 - N) * (15000)) \Pr(N) = 185000$

3.51
$P(5th) = .7^4 * .3 = .072$ (assuming very large pool)

3.63
$E(1sthead) = \sum_{N=0}^{\infty} (N * (\frac{1}{2})^N) = x \frac{\partial}{\partial x} (\frac{1}{1-x})|_{x=.5} = \frac{x}{(1-x)^2}|_{x=.5} = 2$

3.88
$P(Y = 1) = 2/6 * 4/5 * 3/4 * 3 = 3/5$
$P(Y \geq 1) = 2/6 + 4/6 * 2/5 + 4/6 * 3/5 * 2/4 = 1/3 + 4/15 + 1/5 = 12/15 = 4/5$
$P(Y \leq 1) = 1 - P(Y \geq 1) + P(Y = 1) = 1 - 4/5 + 3/5 = 4/5$

3.95 a
$P(all5ok) = 18/20 * 17/19 * 16/18 * 15/17 * 14/16 = .5526$

3.95 b
$P(N_{ok} = 5) = .5526$
$P(N_{ok} = 4) = 5 * \frac{18*17*17*15*2}{20*19*18*17*16} = .3947$
$P(N_{ok} = 3) =_5 C_2 * \frac{18*17*16*2*1}{20*19*18*17*16} = .0526$

$E(Time) = 5 * P(5) + 14 * P(4) + 23 * P(3) = 9.4986$
$\sigma^2(Time) = (5-9.4984)^2 * P(5) + (14-9.4986)^2 * P(4) + (23-9.4986)^2 * P(3) = 28.7692$
$\sigma(time) = 5.3637$

3.97
$P(Y = 4) = \frac{2^Y * e^{-2}}{Y!} = .0902$
$P(Y \geq 4) = \sum_{j=4}^{\infty} \frac{2^j * e^{-2}}{j!} = .1429$
$P(Y \leq 4) = .8571$
$P(Y \geq 4 | Y \geq 2) = \frac{P(Y \geq 4)}{P(Y \geq 2)} = .2405$

3.113
$E(R) = E(1600 - 50Y^2) = 1600 - 50 * E(Y^2)$
$Y \sim$ poisson with $\lambda = 2$
and $E(Y^2) - \mu^2 = Var(Y)$
so $\mu = 2$ and $\sigma^2 = 2$ so $E(Y^2) = \mu^2 + \sigma^2 = 6$
so $E(R) = 1300$