Solutions to Basic Mathematical Review

1

Recall: $\mu = 1/N * \Sigma(X_i)$

1.1

$$\begin{aligned}(1/N) * \Sigma(X_i - \mu)^2 &= 1/N * \Sigma({X_i}^2 + \mu^2 - 2\mu X_i) \\ &= 1/N * \Sigma{X_i}^2 + \mu^2 - 2\mu/N * \Sigma(X_i) \\ &= 1/N * \Sigma{X_i}^2 - \mu^2\end{aligned} \tag{1}$$

1.2

Recall: $X = 1/N * \Sigma(X_i)$ and $Y = 1/N * \Sigma(Y_i)$

$$\begin{aligned}\Sigma[(X_i - X)(Y_i - Y)] &= \Sigma[X_iY_i + XY - X_iY - Y_iX] \\ &= \Sigma(X_iY_i) + NXY - NY * (1/N)\Sigma(X_i) - NX * (1/N) * \Sigma(Y_i) \\ &= \Sigma(X_iY_i) + NXY - NYX - NXY \\ &= \Sigma(X_iY_i) - NXY\end{aligned} \tag{2}$$

2

2.1

$dY/dX = \beta$

2.2

$dY/dX = -\beta(1/X^2)$

2.3

$ln(Y) = \alpha + \beta X$
$Y = e^{\alpha+\beta X} = e^{\alpha} * e^{\beta X}$. So:
$dY/dX = e^{\alpha} * \beta * e^{\beta X}$

2.4

$ln(Y) = \alpha + \beta ln(X) = \alpha + ln(X^{\beta})$
$Y = e^{\alpha} * e^{ln(X^{\beta})} = e^{\alpha} * X^{\beta}$
$dY/dX = e^{\alpha} * \beta * X^{\beta-1}$

2.5

$dY/dX = \beta + 2\lambda X$

3

3.1

$\int_a^b \frac{x}{b-a} dx = \frac{x^2}{2(b-a)}|_a^b = 1/2 * (b^2 - a^2)/(b-a) = 1/2 * (b+a)$

3.2

$\int_a^b \frac{x^2}{(b-a)} dx = \frac{x^3}{3(b-a)}|_a^b = 1/3 * (b^3 - a^3)/(b-a) = 1/3 * (b^2 + ab + a^2)$

4

Integration by parts: $\int udv = u * v - \int vdu$
So: $u = x$ and $dv = \lambda e^{-\lambda x} dx$ so $v = -e^{-\lambda x}$
$\int_t^{\infty} \lambda x e^{-\lambda x} dx = [-x * e^{-\lambda x} - 1/\lambda * e^{-\lambda x}]|_t^{\infty} = 0 + 0 + (t + 1/\lambda) * e^{-\lambda t}$

5

$\int_0^1 \int_0^1 xy * (x+y) dx dy = \int_0^1 y * (x^3/3 + yx^2/2)|_0^1 dy = \int_0^1 y * (1/3 + y/2) dy =$
$y^2/6 + y^3/6|_0^1 = 1/3$

6

6.1

$dY/dX * X/Y = \beta * \frac{X}{\alpha+\beta X}$

6.2

$dY/dX * X/Y = -\beta(1/X^2) * X/Y = -\frac{\beta}{\alpha x+\beta}$

6.3

$dY/dX * X/Y = e^{\alpha} * \beta * e^{\beta X} * \frac{X}{e^{\alpha+\beta X}} = \beta X$

6.4

$dY/dX * X/Y = e^{\alpha} * \beta * X^{\beta-1} * \frac{X}{e^{\alpha} X^{\beta}} = \beta$

6.5

$dY/dX * X/Y = (\beta + 2\lambda X) * \frac{X}{\alpha+\beta X+\lambda X^2}$

7

7.1

$e^{tx} \approx 1 + tx + 1/2 * x^2t^2 + 1/6 * x^3t^3$

7.2

$1/i = i^2/i * 1/i^2 = -i$
$e^{itx} \approx 1 + itx - 1/2 * x^2t^2 - i/6 * x^3t^3$

8

$\frac{\partial S}{\partial \alpha} = -2 * \Sigma(Y_i - \alpha - \beta X_i)$
$\frac{\partial S}{\partial \beta} = -2 * \Sigma[(Y_i - \alpha - \beta X_i) * X_i]$

9

$\frac{\partial L}{\partial \mu} = \frac{\Sigma(X_i - \mu)}{\sigma^2}$
$\frac{\partial L}{\partial \sigma^2} = -\frac{N}{2} * \frac{2\pi}{2\pi\sigma^2} + \frac{\Sigma[(X_i - \mu)^2]}{2\sigma^4} = -\frac{N}{2} * \frac{1}{\sigma^2} + \frac{\Sigma[(X_i - \mu)^2]}{2\sigma^4}$

10

$$\mathbf{X'X} = \begin{pmatrix} 3925 & 760518 \\ 760518 & 14708104 \end{pmatrix}$$

$$(\mathbf{X'X})^{-\mathbf{1}} = \begin{pmatrix} 1.7179 & -0.0888 \\ -0.0888 & 0.0046 \end{pmatrix}$$

$$\mathbf{X'y} = \begin{pmatrix} 1926760 \\ 37261880 \end{pmatrix}$$

$$(\mathbf{X'X})^{-\mathbf{1}}\mathbf{X'y} = \begin{pmatrix} 76.6234 \\ -1.4286 \end{pmatrix}$$